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UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

CLASS - 9

Question Paper Code : 4P114

KEY

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B	C	D	D	B	D	B	C	B	A

EXPLANATIONS

MATHEMATICS - 1

01. (B) By substitution x = 6 in LHS

We get RHS

02. (B) We have

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

$$=\frac{1}{3-\sqrt{8}}\times\frac{\left(3+\sqrt{8}\right)}{\left(3+\sqrt{8}\right)}-\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\left(\sqrt{8}+\sqrt{7}\right)}{\left(\sqrt{8}+\sqrt{7}\right)}$$

$$+\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\left(\sqrt{7}+\sqrt{6}\right)}{\left(\sqrt{7}+\sqrt{6}\right)}-\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\left(\sqrt{6}+\sqrt{5}\right)}{\left(\sqrt{6}+\sqrt{5}\right)}$$

$$+\frac{1}{\sqrt{5}-2}\times\frac{\left(\sqrt{5}+2\right)}{\left(\sqrt{5}+2\right)}$$

$$=-\frac{\left(\sqrt{6}+\sqrt{5}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}+\frac{\left(\sqrt{5}+2\right)}{\left(\sqrt{5}\right)^2-(2)^2}$$

$$= \frac{(3+\sqrt{8})}{(9-8)} - \frac{(\sqrt{8}+\sqrt{7})}{(8-7)} + \frac{(\sqrt{7}+\sqrt{6})}{(7-6)} - \frac{(\sqrt{6}+\sqrt{5})}{(6-5)} + \frac{(\sqrt{5}+2)}{(5-4)}$$

$$= 3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$= 3 + 2 = 5$

03. (B) $$\frac{1}{(x^2+4x+3)} - \frac{1}{(x^2+5x+6)}$$

$$= \frac{1}{(x+3)(x+1)} - \frac{1}{(x+2)(x+3)}$$

$$= \frac{(x+2)-(x+1)}{(x+3)(x+1)(x+2)}$$

$$= \frac{x+2-x-1}{(x^2+4x+3)(x+2)}$$

$\therefore$ K = 1

04. (A) Given $a^2 + b^2 + c^2 - 12a + 10b + 14c + 110 = 0$

$a^2 - 12a + b^2 + 10b + c^2 + 14c + 36 + 25 + 99 = 0$

$a^2 - 12a + 36 + b^2 + 10b + 25 + c^2 + 14b + 49 = 0$

$\therefore (a-6)^2 + (b+5)^2 + (c+7)^2 = 0$

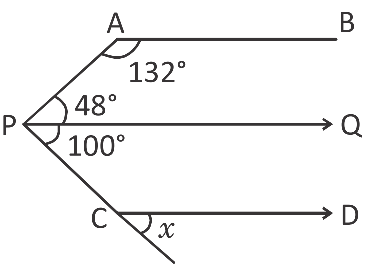
$\therefore$ If $x^2 + y^2 + z^2 = 0$, then $x = 0$, $y = 0$ & $z = 0$

$\therefore$ a – 6 = 0, b + 5 = 0, c + 7 = 0

$\therefore$ a = 6, b = – 5 & c = –7

$\therefore$ a + b + c = 6 + (–5) + (–7) = –6

05. (A)



Construction : PQ || AB

$\angle$BAP + $\angle$APQ = 180°

132° + $\angle$APQ = 180°

$\angle$APQ = 48°

$\Rightarrow \angle$QPC = 148 – 48° = 100°
[$\because$ corresponding angles are equal]

06. (A) Same as above, base = 3

Height = 3

Area $= \frac{1}{2} \times 3 \times 4 = 6$ sq.units

07. (D) Given a = b = c = 8 cm

$$s = \frac{a+b+c}{2} = \frac{24 \text{ cm}}{2} = 12 \text{ cm}$$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{12(12-8)(12-8)(12-8)} \text{ cm}^2$$

$$= \sqrt{3 \times 4 \times 4 \times 4 \times 4} \text{ cm}^2$$

$$= 4 \times 4\sqrt{3} \text{ cm}^2 = 16\sqrt{3} \text{ cm}^2$$

08. (C) Construction: Draw CE || AD and CF $\perp$ AB

$\therefore$ AECD is a parallelogram.

$\therefore$ CE = AD = 6 cm and AE = DC = 10 cm

$\therefore$ EB = 20 cm – AE = 10 cm

In ΔBCE, $BC^2 + CE^2 = 8^2 + 6^2 = 10^2 = BE^2$

$\therefore \angle$BCE = 90°

$\therefore$ Area of $\Delta BCE = \frac{1}{2} \times BC \times EC$

$= \frac{1}{2} \times 6 \text{ cm} \times 8 \text{ cm}$

$= 24 \text{ cm}^2$

But area of $\Delta BCE = \frac{1}{2} \times BC \times CF$

$24 \text{ cm}^2 = \frac{1}{2} \times 10 \text{ cm} \times CF$

$$CF = \frac{24\text{cm}^2 \times 2}{10 \text{ cm}} = 4.8 \text{ cm}$$

Area of the trapezium ABCD $= \frac{1}{2}$ CF (AB + CD)

$= \frac{1}{2} \times 4.8 \ (20 + 10) \text{ cm}^2$

$= 2.4 \times 30 \text{ cm}^2$

$= 72 \text{ cm}^2$

09. (C) Volume of one laddoo of radius 5 cm

$$=\left\{\frac{4}{3}\pi\times(5)^3\right\}cm^3=\left(\frac{500\pi}{3}\right)cm^3$$

From this laddoo, let n laddoos of radius 2.5 cm can be made.

Volume of one laddoo of radius 2.5 cm

$$=\left\{\frac{4}{3}\pi\times\left(\frac{5}{2}\right)^3\right\}cm^3=\left(\frac{125\pi}{6}\right)cm^3$$

$$\therefore n=\frac{\text{volume of one laddoo of radius 5cm}}{\text{volume of one laddoo of radius 2.5 cm}}$$

$$=\left(\frac{500\pi}{3}\times\frac{6}{125\pi}\right)=8.$$

Hence, the required number of laddoos is 8

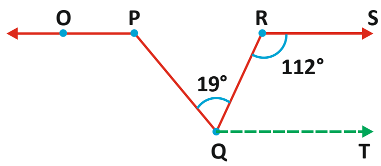
10. (C) $\angle COA = 2\angle CBA = 2 \times 30° = 60°$

In $\angle COA$, $\angle COA = 60°$ and CO = OA

COA is an equilateral triangle.

$\therefore \angle CAO = 60°$

11. (C) Const draw QT||RS



$\angle QRS + \angle RQT = 180°$ [$\because$ If the lines are parallel the sum of interior angles lie same to the transversal are supplimentary]

$\therefore$ $\angle RQT = 180° - 112°$

$\angle RQT = 68°$

But $\angle OPQ = \angle PQT$

[$\because$ Alternative angles]

$\therefore$ $\angle OPQ = \angle PQR + \angle RQT$

$= 19° + 68° = 87°$

12. (D) $$\frac{1}{\sqrt{11}-3}=\frac{1}{\sqrt{11}-3}\times\frac{\sqrt{11}+3}{\sqrt{11}+3}$$

$$=\frac{\sqrt{11}+3}{(\sqrt{11})^2-3^2}=\frac{\sqrt{11}+3}{11-9}=\frac{\sqrt{11}+3}{2}$$

Similarly,

$$\frac{1}{3-\sqrt{7}}=\frac{3+\sqrt{7}}{2}$$

$$\frac{1}{\sqrt{7}-\sqrt{5}}=\frac{\sqrt{7}+\sqrt{5}}{2}$$

$$\frac{1}{\sqrt{5}-\sqrt{3}}=\frac{\sqrt{5}+\sqrt{3}}{2}$$

$$\therefore \frac{\sqrt{5}+\sqrt{3}}{2}<\frac{\sqrt{7}+\sqrt{5}}{2}<\frac{3+\sqrt{7}}{2}<\frac{\sqrt{11}+3}{2}$$

i.e., $$\frac{1}{\sqrt{5}-\sqrt{3}}<\frac{1}{\sqrt{7}-\sqrt{5}}<\frac{1}{3-\sqrt{7}}<\frac{1}{\sqrt{11}-3}$$

$$\Rightarrow \sqrt{5}-\sqrt{3}>\sqrt{7}-\sqrt{5}>3-\sqrt{7}>\sqrt{11}-3$$

13. (B) $\angle PGH = \angle PGH = 85°$

[corresponding angles]

$\angle QGH = 180° - \angle PGH = 180° - 85° = 95°$

$\angle QGC = x + \angle QGH$

$115° = x + 95°$

$115° - 95° = x$

$x = 20$

14. (D) Divide the second equation by 2

$2u - 3v = 0$

Now add to the first equation

$(2u + 3v) + (2u - 3v) = 2 + 0$

$4u = 2$

$$u=\frac{1}{2}$$

Substitute into $2u - 3v = 0$

$$2\left(\frac{1}{2}\right)-3v=0$$

$1 - 3v = 0$

$$v=\frac{1}{3}$$

15. (B) $x + y = 0$

$(-2) + 2 = 0$

$0 + 0 = 0$

$2 + (-2) = 0$

16. (D) Extend BA up to F such that BA line meets EC at F

$\angle BAE = \angle E + \angle AFE$

$= \angle E + \angle ECD$

[$\therefore$ $\angle AFE = \angle ECD$ corresponding angles]

$\angle BAE - \angle ECD = \angle E = \angle FEC$

17. (C) In a cyclic quadrilateral ABCD

$\angle A + \angle C = 180°$

$2x + 44 + 4x + 64° = 180°$

$6x + 108° = 180°$

$6x = 180° - 108°$

$x = \frac{72°}{6} = 12°$

18. (B) Const:- Extand GH Up to 5

$\angle AIH = 70°$

[$\because$ corresponding angles]

$\therefore$ $\angle AIJ = 180° - 70° = 110°$

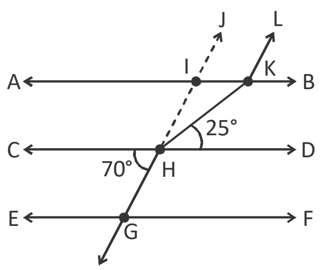
$\Rightarrow$ $\angle IKL = \angle AIJ = 110°$

[$\because$ corresponding angles]

$\angle IKH = \angle KHD = 25°$

[$\because$ alternative angles]

$\angle HKL = \angle HKI + \angle IKL = 25° + 110° = 135°$



19. (D) Given $\pi r^2 h = 23{,}100\ cm^3$

$\frac{22}{7} \times r^2 \times 24\ cm = 23{,}100\ cm^3$

$r^2 = 23{,}100 \times \frac{7}{22} \times \frac{1}{24}$

$= \frac{25 \times 7 \times 7}{4}\ cm^2$

$r = \sqrt{\frac{25 \times 7 \times 7}{4}}\ cm = \frac{5 \times 7}{2}\ cm$

TSA of cylender = 2pr(h + r)

$= 2 \times \frac{22}{7} \times \frac{35}{2}(24 + 17.5)\ cm^2$

$= 110 \times 41.5 = 4565$

20. (C) Side of each tile are 36 cm, 29 cm & 25 cm

$\therefore$ $S = \frac{a + b + c}{2} = \frac{(36 + 29 + 25)}{2}\ cm$

$S = 45\ cm$

Area of each tile

$= \sqrt{S(s - a)(s - b)(s - c)}$

$= \sqrt{45 \times 9 \times 16 \times 20}\ cm^2$

$= \sqrt{5 \times 9 \times 9 \times 4 \times 4 \times 5 \times 2 \times 2}\ cm^2$

$= 5 \times 9 \times 4 \times 2\ cm^2$

$= 360\ cm^2$

$\therefore$ Area of 16 tiles $= 360\ cm^2 \times 16$

$= 5760\ cm^2$

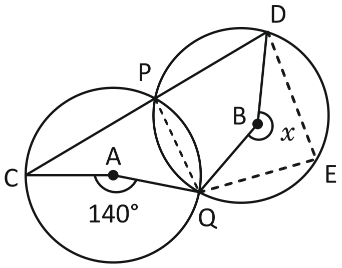
$\therefore$ Total cost for polishing

$= 5760\ \cancel{cm^2} \times \frac{80\ paise}{1\ \cancel{cm^2}}$

$= ₹\ 576\cancel{0} \times \frac{8\cancel{0}}{10\cancel{0}}$

$= ₹\ 4608$

21. (C) Construction:



Join PQ–Notice a point 'E' on the circumference of circle of centre B. Join EQ & DE

$\angle CPQ = \dfrac{\angle CAQ}{2} = \dfrac{140^\circ}{2} = 70^\circ$

$\therefore\ \angle QPD = 180^\circ - \angle CPQ = 110^\circ$

PQED is a cycle quadrilateral

$\therefore\ \angle QED + \angle QPD = 180^\circ$

$\therefore\ \angle QED + 110^\circ = 180^\circ$

$\therefore\ \angle QED = 70^\circ$

$\therefore\ \angle QBD = 2\angle QED = 140^\circ$

$\therefore\ x = 360^\circ - \angle QBD = 360^\circ - 140^\circ = 220^\circ$

22. (D) $\alpha\beta = \left(\dfrac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\dfrac{-b-\sqrt{b^2-4ac}}{2a}\right)$

$= \dfrac{(-b)^2 - \left(\sqrt{b^2-4ac}\right)^2}{4a^2}$

$= \dfrac{b^2 - (b^2-4ac)}{4a^2}$

$= \dfrac{b^2 - b^2 + 4ac}{4a^2} = \dfrac{4ac}{4a^2} = \dfrac{c}{a}$

23. (C) Total surface area = CSA of a cylinder + 2CSA of the hemisphere

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi r(h + 2r)$

$= 2 \times \dfrac{22}{7} \times 35\text{ cm}(100+70)\text{ cm}$

$= 37{,}400\text{ cm}^2$

24. (C) $S = \dfrac{a+b+c}{2} = \dfrac{61\text{ cm} + 102\text{ cm} + 109\text{ cm}}{2}$

$= \dfrac{272}{2}\text{ cm} = 136\text{ cm}$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{136 \times 75 \times 34 \times 27}\ \text{cm}^2$

$= \sqrt{8 \times 17 \times 5 \times 5 \times 3 \times 2 \times 17 \times 3 \times 9}\ \text{cm}^2$

$= \sqrt{16 \times 17^2 \times 5^2 \times 9^2}\ \text{cm}^2$

$= 4 \times 17 \times 5 \times 9\text{ cm}^2$

$= 3060\text{ cm}^2$

25. (B) Given $\pi r(l + r) = 7920\text{ cm}^2$

$\pi rl + \pi r^2 = 7920\text{ cm}^2$

given $\pi rl = 4070\text{ cm}^2$

$\therefore\ \pi rl + \pi r^2 - \pi rl = (7{,}920 - 4070)\text{cm}^2$

$\dfrac{22}{7} \times r^2 = 3850$

$r^2 = 3850 \times \dfrac{7}{22}$

$r = \sqrt{7 \times 5 \times 5 \times 7\text{ cm}^2}$

$r = 35\text{ cm}$

Given $\dfrac{22}{7} \times 35\text{ cm} \times l = 4070\text{ cm}^2$

$l = \dfrac{4070\text{ cm}^2}{22 \times 5\text{ cm}} = 37\text{ cm}$

$h = \sqrt{l^2 - r^2}$

$= \sqrt{37^2 - 35^2}$

$= \sqrt{(37+35)(37-35)}$

$= \sqrt{72 \times 2}$

$h = \sqrt{144}$

$h = 12\text{ cm}$

26. (C) equ (1) + (2) + (3)

$\Rightarrow 12x + 12y + 12z = 38 + 35 + 35 = 108$

$12(x+ y + z) = 108$

$x+ y + z = \frac{108}{12} = 9$

27. (B) Volume of cube = $(7cm)^3 = 343cm^3$

Diameter of cone = side of cube = 7cm

$\therefore$ r = $\frac{7cm}{2}$ & h = 7cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7cm^3$

= 89.83 cm^3

wastage of wood = 343 cm^3 – 89.83cm^3 = 253.17cm^3

waste wood percentage

$= \frac{253.17cm^3}{343cm^3} \times 100$

= 73.81%

28. (C) Given ∠A = 42° + ∠B &

∠C = ∠B – 21°

But ∠A + ∠B + ∠C = 180°

42° + ∠B + ∠B + ∠B – 21° = 180°

3∠B + 21° = 180°

3∠B = 180° – 21° = 159°

$\angle B = \frac{159^\circ}{7} = 53^\circ$

But ∠A = 42° + ∠B = 42 + 53° = 95°

29. (C) From the given figure, we have

AD = BC

$\Rightarrow$ AD – CD = BC – CD

[Equal are subtracted from equals]

$\Rightarrow$ AC = BD

30. (D) Given $f(x) = x^2+7x - 18$

$x^2 + 7x - 18$

$x^2 + 9x - 2x - 18$

$x(x + 9) - 2(x + 9)$

$(x + 9)(x - 2)$

$\therefore$ f(–9) = 0 and f(–2) = 0

$\therefore$ p = – 9 and q = 2

$\therefore$ p + q = –9 + 2 = –7

MATHEMATICS - 2

31. (C,D)

$2^{2x} + 1 = 17 \times 2^x - 8$

Let $y = 2^x$, then

$y^2 + 1 = 17y - 8$

$2y^2 - 17y + 8 = 0$

Solving quadratic

$y = \frac{17 \pm \sqrt{289-64}}{4} = \frac{17 \pm 15}{4}$

$\Rightarrow y = 8, 0.5$

So, $2x = 8 \Rightarrow x = 3$, or $2x = 0.5$

$x = -1$

32. (A,B,D)

In ΔABE, $\frac{\angle A}{2} + \frac{\angle B}{2} + \angle E = 180^\circ$

$\frac{\angle A + \angle B}{2} + \angle E = 180^\circ$

$\frac{180^\circ}{2} + \angle E = 180^\circ$

90 + ∠E = 180°

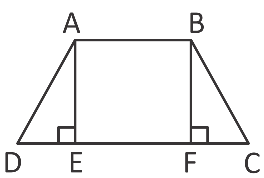
∠E = 180° – 90°

∠E = 90°

$\Rightarrow$ option 'A' is true.

similarly we can prove the option 'B' also

Construction:



Draw AE ⊥ CD and true

BF ⊥ CD Join AC & BD

$\Delta AED \cong \Delta BFC$ ($\therefore$ RHS Congruency)

$\therefore \angle C = \angle D$ ($\therefore$ CPCT)

$\Delta ACD \cong \Delta BDC$ ($\therefore$ SAS Congruency)

$\therefore$ AC = BD

33. (B,C)

Both options lie on the line $x + y = 0$

34. (A,C,D)

In a rectangle, square and isosceler tapezium opposite angles are supplimentary.

$\therefore$ Rectangle, square and isosceles trapezium are circlic quadrilateral.

35. (B,C,D)

$\sqrt{12} = \sqrt{2\times2\times3} = 2\sqrt{3}$

$2\sqrt{3} \times \sqrt{3} = 2 \times 3 = 6$ which a rational number

$2\sqrt{3} \times \sqrt{27} = 2\sqrt{3} \times 3\sqrt{3}$

$= 6 \times 3 = 18$ which a rational number

$2\sqrt{3} \times \sqrt{75} = 2\sqrt{3} \times 5\sqrt{3}$

$= 10 \times 3 = 30$

Which a rational number

$\sqrt{3}$, $\sqrt{27}$ & $\sqrt{75}$

<u>REASONING</u>

36. (D) R × T → R is father of T

P + R → P is brother of R → So, P is T's uncle

37. (A) 8280 [Pattern : × 3, × 4, × 5, × 6]

38. (A) One drinks water when he is thirsty.

since, 'water' drink 'light'when he is thirsty there. So, the answer is 'light'.

39. (C) In the first row, (28/7) × 5 = 20.

In the second row, (84/12) × 5 = 35.

In the third number, missing number = (45/ 9) × 5 = 25.

40. (C) The blank face in the question is the one above the centre in the cube net.

The face below the centre is the diagonal–shaded face.

When you fold the net into a cube, the top and bottom faces become opposite to each other.

So the face opposite the blank face is the diagonal-shaded one → Option (C).

41. (B) Given that,

F is 1 km to the west of D.

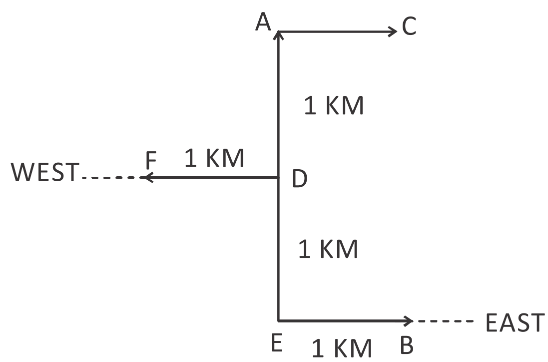
B is 1 km to the east of E.

A is 2 km to the north of E.

C is 1 km to the east of A.

D is 1 km to the south of A.

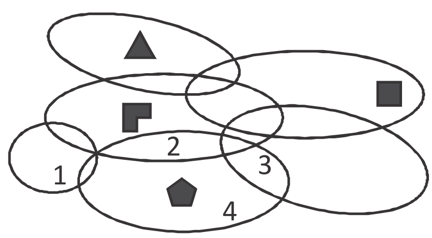
Now,



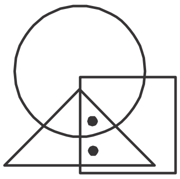
Three villages A, D, E are on same line

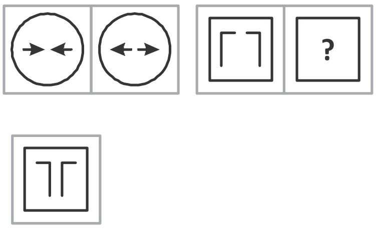
Hence option (B) is correct

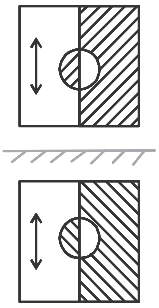
42. (C) All shapes are enclosed in an area whose number of sides are equal to the number of circles which the enclosed area is made up of.



Only in option C, which has a shape with 5 sides and it is enclosed in an area with 4 circles.

43. (D) 

44. (D) 

45. (B) 

CRITICAL THINKING

46. (D) Total steps = 2023 → black steps = every 3rd → 674 black steps.

Right foot lands on every other step.

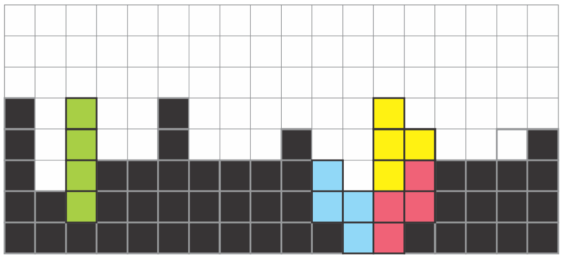
Half of the black steps fall on the right foot → 674 ÷ 2 = 337.

47. (B) We have: "All subjects are tough." That means every subject is in the set "tough."

Conclusions: I. Some subjects are easy. — No: from "all subjects are tough" we cannot infer any subjects are easy (unless "easy" overlaps with "tough," which is not given). So I does not follow. II. Some easy are tough. — Also does not follow: while every subject is tough, we are not told that any "easy" things exist or overlap with "subjects." Without information that some easy things exist and are subjects (or overlap), we cannot conclude "some easy are tough."

Therefore neither conclusion follows. Answer: (B) Neither conclusion I nor II follows.

48. (C)



49. (B) From the data: Q is the Commando and T is the Accountant, so both live in Kolkata. S and U live in Mumbai, P lives in Delhi → the remaining city Chennai must be for R.

The Teacher lives in Chennai, therefore R is the Teacher and lives in Chennai.

Checking options:

Option A : Not definite

Option B : Always true

Option C : False (Commandos do not marry)

Option D : Not necessarily true

50. (A) Statement I:

3rd Saturday is 17th ⇒ Saturdays are 3, 10, 17, 24

⇒ 14th is Wednesday → sufficient.

Statement II:

"2nd day from last is Tuesday" → month length not known

⇒ cannot find 14th → not sufficient.